SAT

Subject Test Math IIC

Strategy Notes

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Test Basics

- Scoring: 200 – 800
- Questions can be: easy, medium, and difficult
- All questions are worth the same amount
  - Right: +1 point
  - Blank: 0 points
  - Wrong: -1/4 point
- Timing: 60 minutes
- Questions: 50 multiple choice
- Graphing calculators allowed

Reference Information

- Volume of a right circular cone with radius *r* and height *h*:

$$V = \frac{1}{3}\pi r^2 h$$

- Volume of a sphere with radius *r*:

$$V = \frac{4}{3}\pi r^3$$

- Volume of a pyramid with base area *B* and height *h*:

$$V = \frac{1}{3}Bh$$

- Surface Area of a sphere with radius *r*:

$$S = 4\pi r^2$$

General Strategies

- Pacing: Focus your time on the easy and medium questions (no careless mistakes!)
- Double pass: Not sure, skip it and come back later
- Use POE: Eliminate 2 or more, then GUESS
- Your calculator is only as smart as you are
- Write only what you need to

Additional SAT Topics

- Logarithms
- Matrices
- Vectors
- Exponential functions
- Trigonometric functions
- Piecewise functions
- Recursive functions
- Hyperbolas
- Polar coordinates
- Cosines
- Sines
- Standard deviation
- Least squares regression
- Irrational numbers

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Algebra

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Logarithms

- The three equations below are equivalent:

$$log_a x = b$$
$$a^b = x$$
$$\sqrt[b]{x} = a$$

- Example:

$$log_4 16 = 2$$
$$4^2 = 16$$
$$\sqrt[2]{16} = 4$$

Logarithm Rules

- When logarithms of the same base are multiplied, the base remains the same and the exponents can be added:

$$log_x jk = log_x j + log_x k$$

$$log_2 4 + log_2 3 = log_2 12$$

Logarithm Rules

- When a logarithm is raised to a power, the exponent can be brought in front and multiplied by the logarithm:

$$log_x c^n = n \times log_x c$$

$$logx^4 = 4logx$$

Logarithm Rules

- When logarithms of the same base are divided, the exponents must be subtracted:

$$log_x \frac{j}{k} = log_x j - log_x k$$

$$log\ \frac{1}{2} = log\ 1 - log\ 2$$

Natural Logarithms

- A natural logarithm (*ln*) has base *e* and will most likely be used in problems for growth and decay
- Three equivalent equations:

$$ln\, x = b$$
$$log_e x = b$$
$$e^b = x$$

Binomial Theorem

- Notice that patterns emerge when we raise the binomial $(a + b)$ to consecutive powers:

$$(a + b)^0 = 1$$
$$(a + b)^1 = a + b$$
$$(a + b)^2 = a^2 + 2ab + b^2$$
$$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$
$$(a + b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$

Binomial Theorem

- Looking at these patterns, we can make predictions about the expansion of $(a + b)^n$.

1. There are $n + 1$ terms in the expansion. For example, when the exponent, n, is 4, there are 5 terms.
2. The power to which a is raised decreases by one each term, beginning with n and ending with 0. For example, if $n = 4$, then a in the second term is raised to the third power.
3. Subsequently, the exponent of b increases by one each term, beginning with 0 and ending with n. If $n = 4$, then b in the second term is raised to the first power.
4. The sum of the exponents for each term of the expansion is n.

Binomial Theorem

Example:

What is the thirteenth expansion of $(x + y)^{17}$?

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Plane Geometry

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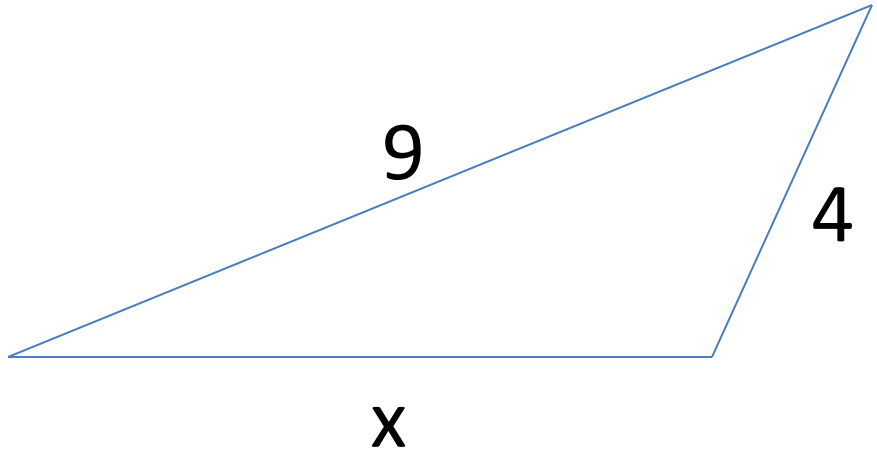
Triangles

- The length of a side of a triangle is less than the sum of the lengths of the other two sides and greater than the difference of the lengths of the other two sides.

$5 < x < 13$



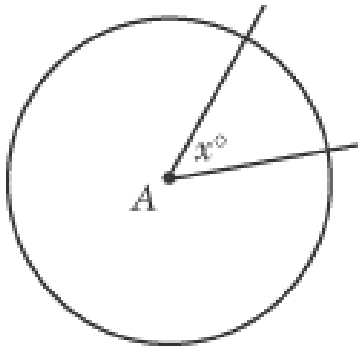
Polygons

- The sum of the interior angles of a polygon with n sides is $(n - 2)180º$. So, for example, the sum of the interior angles of an octagon is $(8 - 2)180º = 6(180º) = 1080º$.
- The sum of the exterior angles of any polygon is 360º.

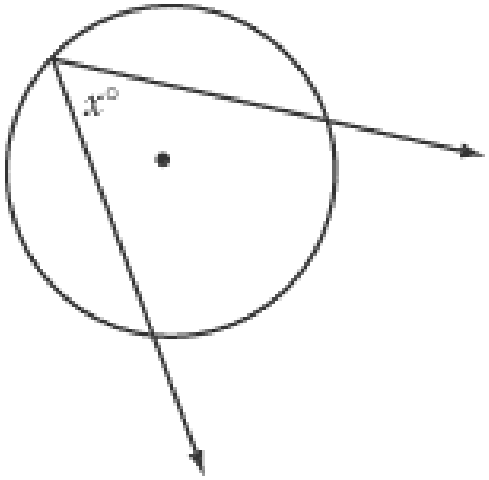
Number of Sides	Name
3	triangle
4	quadrilateral
5	pentagon
6	hexagon
7	heptagon
8	octagon
9	nonagon
10	decagon
12	dodecagon
n	n-gon

Circles

- The degree of the circle cut by a **central angle** is equal to the measure of the angle. If a central angle is 25º, then it cuts a 25º arc in the circle.

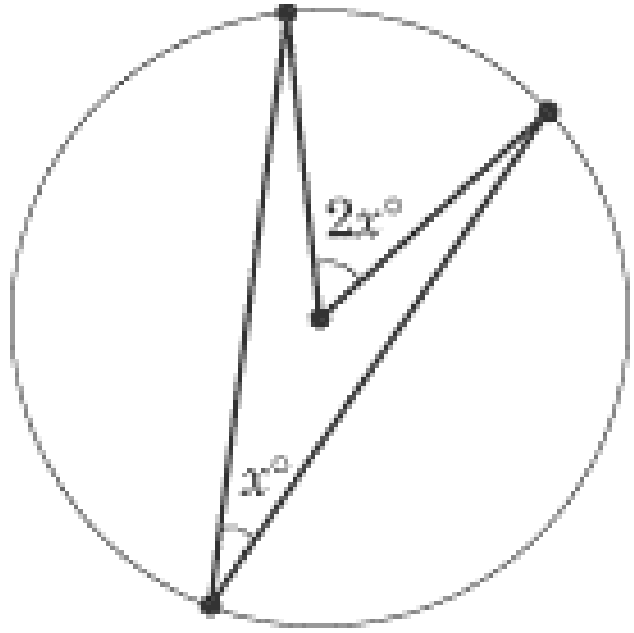


- An inscribed angle will always cut out an arc in the circle that is twice the size of the degree of the inscribed angle. If an inscribed angle has a degree of 40º, it will cut an arc of 80º in the circle.



Circles

- If an inscribed angle and a central angle cut out the same arc in a circle, the central angle will be twice as large as the inscribed angle.



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Solid Geometry

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Solids

Example:

If you double the length of the side of a square, by how much do you increase the area of that square?

Solids

Example:

If a sphere’s radius is halved, by what factor does its volume decrease?

Solids

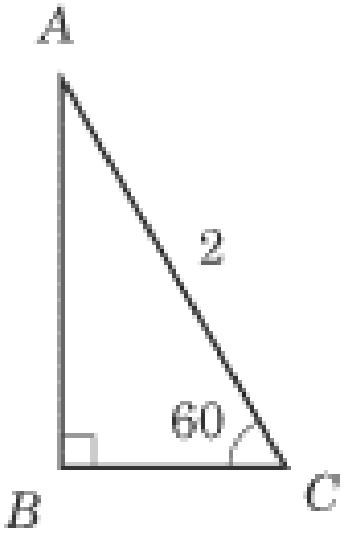
Example:

A cube is inscribed in a cylinder. If the length of the diagonal of the cube is $4\sqrt{3}$ and the height of the cylinder is 5, what is the volume of the cylinder?

Solids

Example:

What is the surface area of the geometric solid produced by the triangle below when it is rotated 360° about the axis AB?



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Coordinate Geometry

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Distance Formula

Example:

Calculate the distance between (4, -3) and (-3,8).

Midpoint Formula

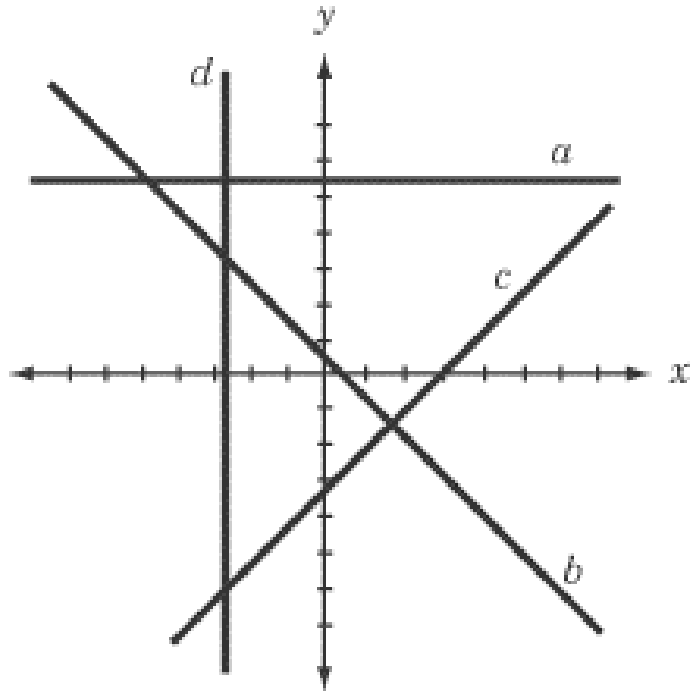
Example:

What is the midpoint of the line segment whose endpoints are (6,0) and (3,7)?

Slope

Example:

Of the four lines pictured below, one has a slope of 0, one has a slope of 1, another has a slope of -1, and another has an undefined slope. Decide which is which.



Slope

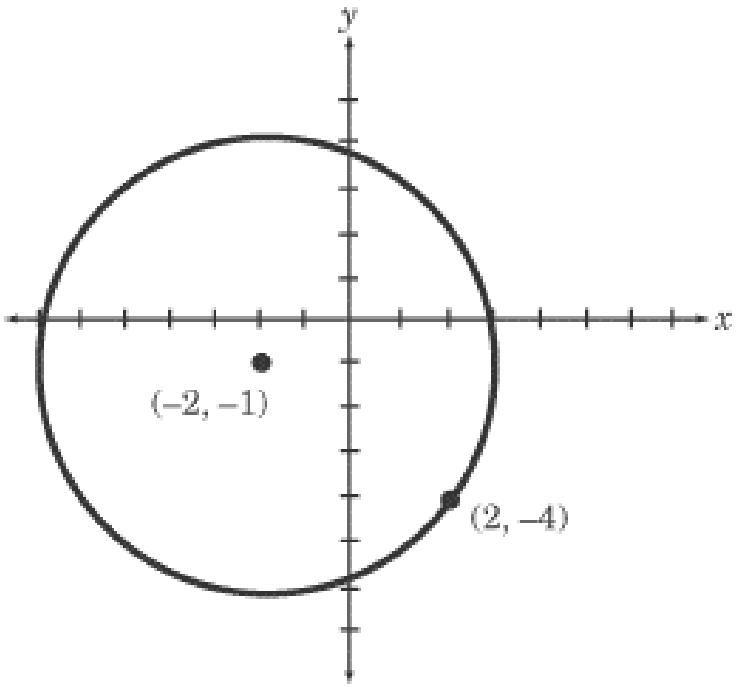
Example:

What is the slope-intercept form of the equation of the line that contains the points (5, 3) and (-1, 8)?

Graphs and Equations

Example:

What is the equation of the circle pictured below?



Graphs and Equations

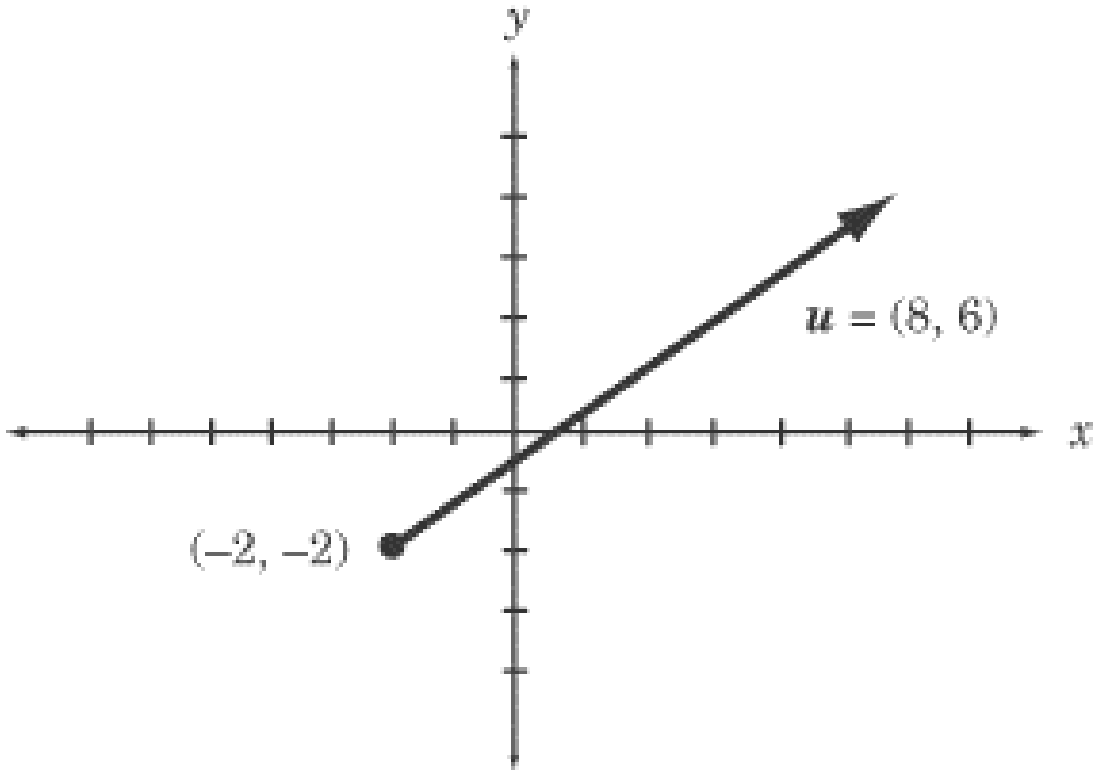
Example:

What are the coordinates of the center and vertices of an ellipse given by the following equation?

$$\frac{(x-2)^2}{16} - \frac{(y+5)^2}{36} = 1$$

Vectors

- Initial point?
- Terminal point?
- Length of vector?



Vectors

- Adding vectors

$$a = (2, 7) \text{ and } b = (-5, 2)$$

$$a + b = (2 + -5, 7 + 2) = (-3, 9)$$

- Subtracting vectors

$$a - b = a + (-b) = (2 - (-5), 7 - 2) = (7, 5)$$

- Multiplying by a scalar

$$u = (3, 4) \rightarrow cu = (3c, 4c)$$

Vectors

Example:

If u = (1, -5) and v = (4, 2), what is 3u – 2v?

Coordinate Space

Example:

What is the distance between points (4, 1, -5) and (-3, 3, 6)?

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Functions

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Functions

Example:

If $f(x) = x^2 - 3$, what is $f(5)$?

Example:

If $f(x) = \frac{3x}{4-x}$, what is $f(x+1)$?

Function Rules

	Rule	Example
Addition	$(f+g)(x) = f(x)+g(x)$	If f(x) = sin x, and g(x) = cos x: (f + g)(x) = sin x + cos x
Subtraction	$(f-g)(x) = f(x)-g(x)$	If f(x) = x2 + 5 and g(x) = x2 + 2x + 1: (f – g)(x) = x2 + 5 – x2 – 2x – 1 = –2x + 4
Multiplication	$(f\times g)(x) = f(x)\times g(x)$	If f(x) = x and g(x) = x3 + 8: (f × g)(x) = x × (x3 + 8) = x4 + 8x
Division	$\frac{f}{g}(x) = \frac{f(x)}{g(x)}, g(x)\neq 0$	If f(x) = 2 cos x, and g(x) = 2 sin2 x: $(f+g)(x) = \frac{2\cos x}{2\sin^2 x} = \frac{\cos x}{\sin^2 x}$

Compound Functions

Example:

Suppose $h(x) = x^2 + 2x$ and $j(x) = |\frac{x}{4} + 2|$. What is $j(h(4))$?

Compound Functions

Example:

Suppose $f(x) = 3x + 1$ and $g(x) = \sqrt{5x}$. What is $g\big(f(x)\big)$?

Inverse Functions

Example:

What is the inverse of $f(x) = 3x + 2$?

Inverse Functions

Example:

What is the inverse of the function $f(x) = x\sqrt{\frac{2x^2-3}{5}}$?

Inverse Functions

Example:

Is the inverse of $f(x) = x^2$ a function?

Inverse Functions

Example:

What is the inverse of $f(x) = 2|x - 1|$, and is it a function?

Domain and Range

Example:

What is the domain of $f(x) = \frac{2}{(x^2+5x+6)}$?

Domain and Range

Example:

What is the domain of $f(x) = \frac{2\sqrt{x-4}}{x-7}$?

Domain and Range

Example:

What is the range of $\frac{|x-3|}{2}$?

Domain and Range

Example:

What is the range of $\frac{\sqrt{|x-6|+4}}{2}$?

Domain and Range

Example:

What is the range of $f(x) = \frac{-3x^2+2}{2}$?

Domain and Range

Example:

$f(x) = 2x^2 + 4$ for $-3 < x < 5$. What is the range of f?

Identifying the Graphs of Polynomials

- Roots: Set the function equal to 0 and solve for x
- Degree: The graph produced by an n-degree function can have as many as n – 1 “bumps” (extreme points)
- End behavior:
  - If degree of polynomial is even, behavior same as $+\infty$ and $-\infty$
  - If degree of polynomial is odd, behavior opposite as $+\infty$ and $-\infty$
- Function symmetry:
  - Even functions are symmetrical with respect to the y-axis
  - Odd functions are symmetrical with respect to the origin
  - No function can have symmetry across the x-axis

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Trigonometry

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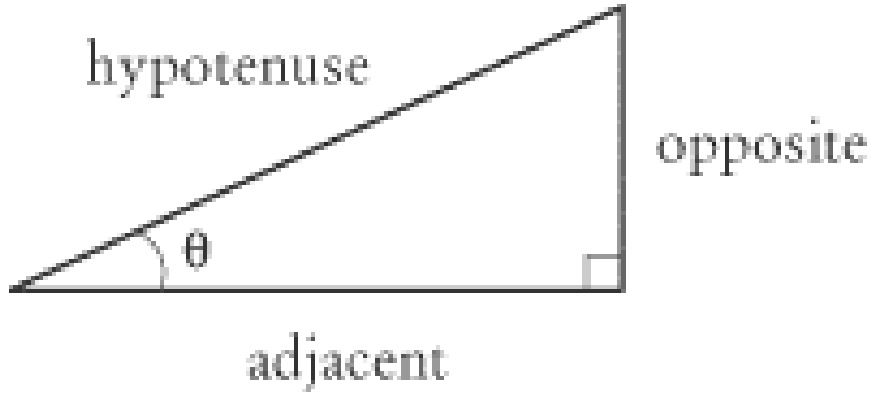
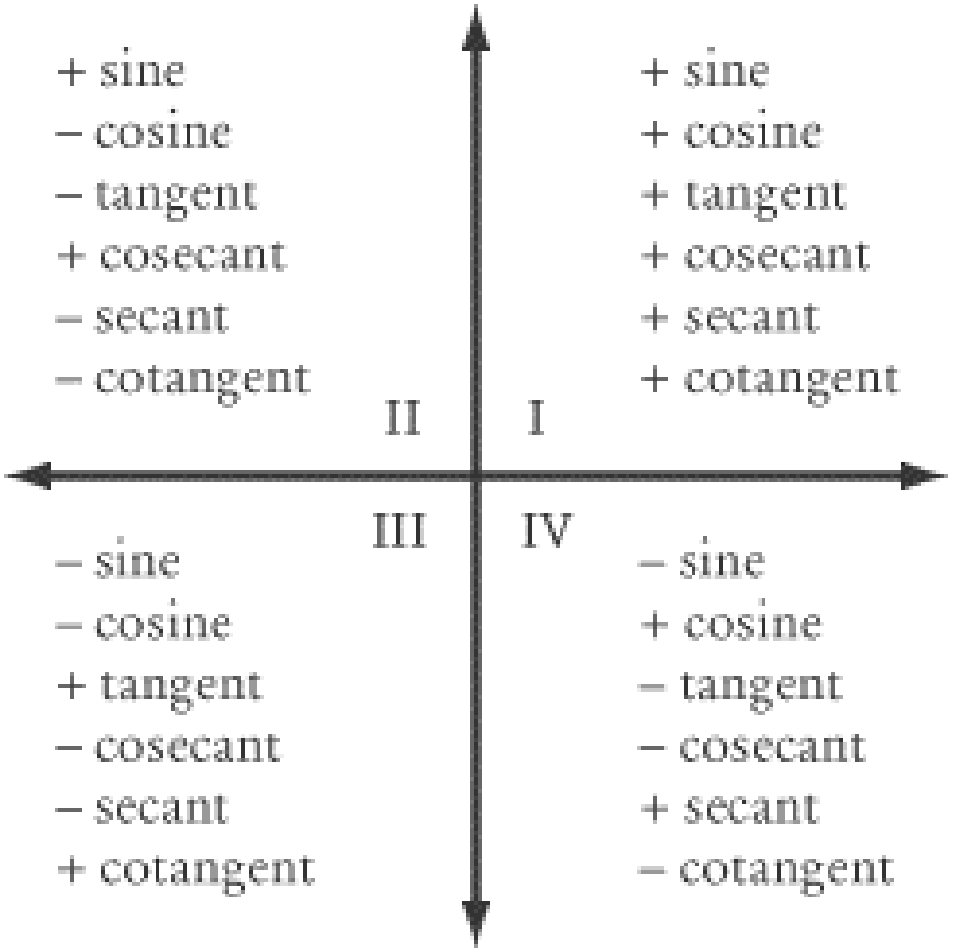
Basic Functions

- $\sin\theta = \frac{opposite}{hypotenuse}$
- $\cos\theta = \frac{adjacent}{hypotenuse}$
- $\tan\theta = \frac{opposite}{adjacent}$



Signs of Trigonometric Functions



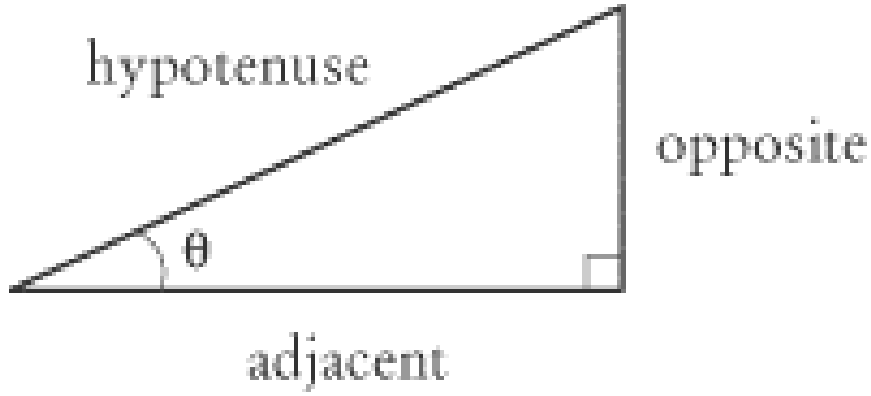
Signs of Trigonometric Functions

Example:

If the value of $\sin -\theta$ is 0.5, what is the value of $\sin \theta$?

Other Functions

- $csc\,\theta = \frac{hypotenuse}{opposite} = \frac{1}{sin\theta}$
- $sec\,\theta = \frac{hypotenuse}{adjacent} = \frac{1}{cos\theta}$
- $\cot\theta = \frac{adjacent}{opposite} = \frac{1}{tan\theta}$



Other Functions

Example:

What is $\frac{sec\theta}{csc\theta}$, if $\theta = 45°$?

Trigonometric Identities

Example:

What is $\frac{cos\theta \times tan\theta}{sin\theta} - cos^2\theta$?

Trigonometric Identities

Example:

What is $tan^2\theta cos^2\theta + 1 - sin^2\theta$?

Trigonometric Identities

Example:

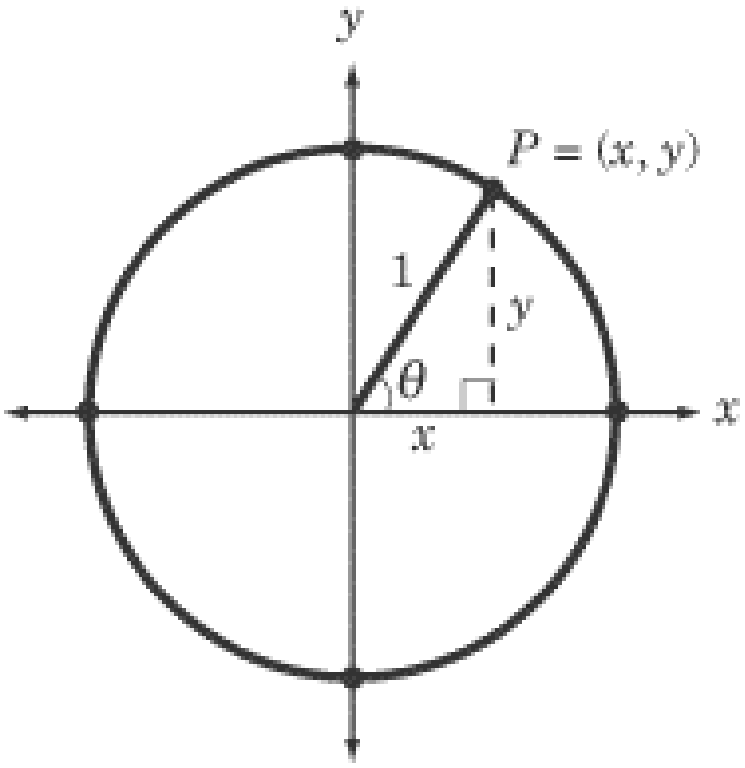
Simplify the following expression: $sin^2x + cos^2x - sec^2x + tan^2x$.

Trigonometric Identities

Example:

If $\sin a = -\cos b = \frac{3}{5}$ and a and b are both in the second quadrant, what is $\cos(a - b)$?

Graphing Trigonometric Functions



$(cos\theta, sin\theta)$

$$\sin\theta = \frac{y}{1} = y$$

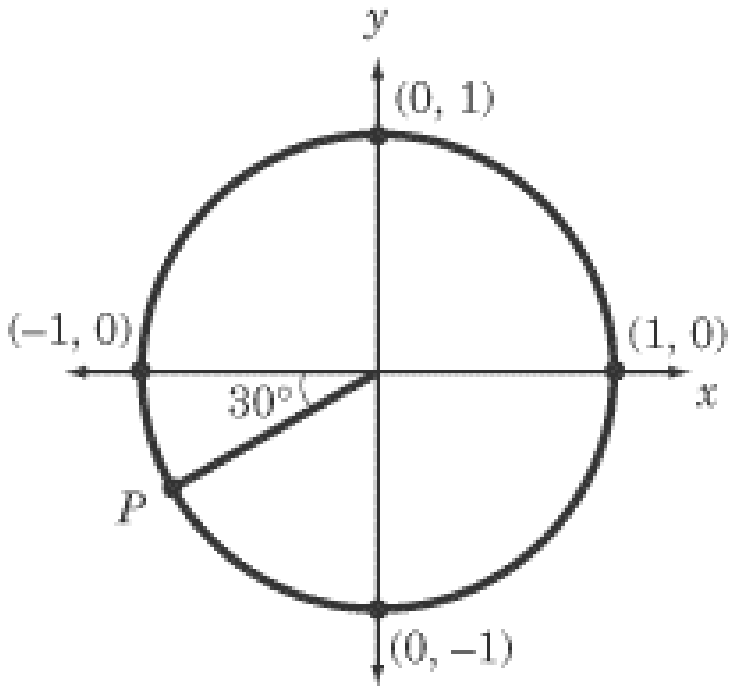
$$\cos\theta = \frac{x}{1} = x$$

$$\tan\theta = \frac{y}{x}$$

Graphing Trigonometric Functions

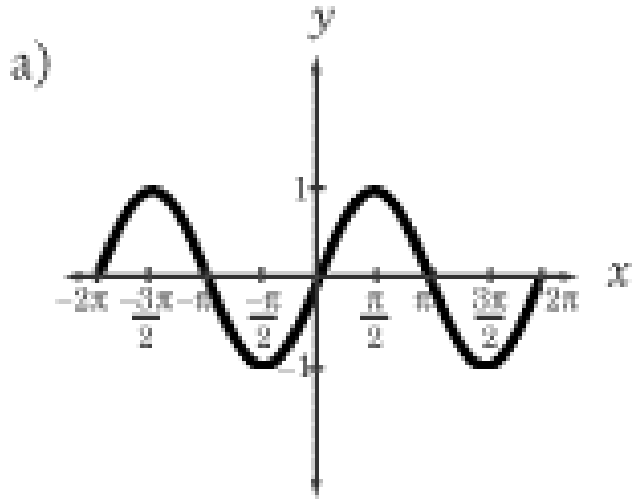
Example:

What are the coordinates of the point P pictured below?

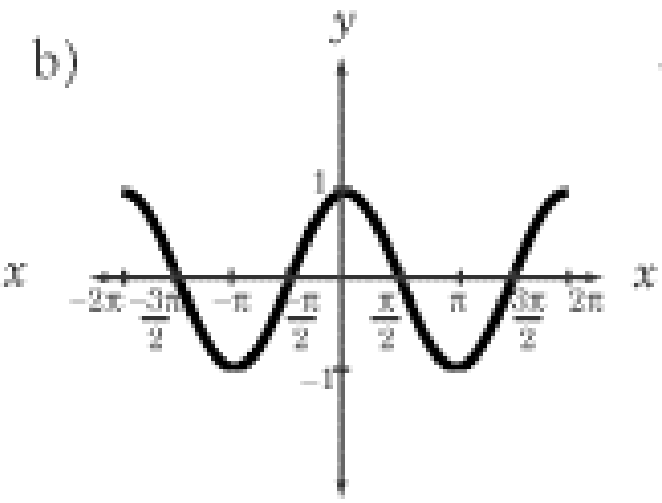


Graphing in the Entire Coordinate Plane

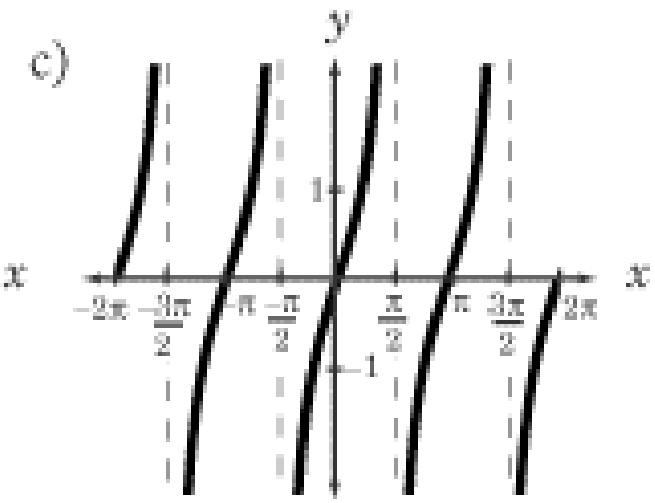
y = sin x



y = cos x



y = tan x



Domain and Range

Function	Domain	Range
$y = \sin(x)$	$-\infty < x < \infty$	$-1 \leq y \leq 1$
$y = \cos(x)$	$-\infty < x < \infty$	$-1 \leq y \leq 1$
$y = \tan(x)$	$-\infty < x < \infty, x \neq n\pi + \pi/2$; n is an integer	$-\infty < y < \infty$
$y = \csc(x)$	$-\infty < x < \infty, x \neq n\pi$; n is an integer	$y \leq -1$ or $y \geq 1$
$y = \sec(x)$	$-\infty < x < \infty, x \neq n\pi + \pi/2$; n is an integer	$y \leq -1$ or $y \geq 1$
$y = \cot(x)$	$-\infty < x < \infty, x \neq n\pi$; n is an integer	$-\infty < y < \infty$

Period and Amplitude

- The period of y = f(bx) is the normal period of f divided by b
- The amplitude of the trigonometric function y = af(x) is |a|
- Tangent has no amplitude

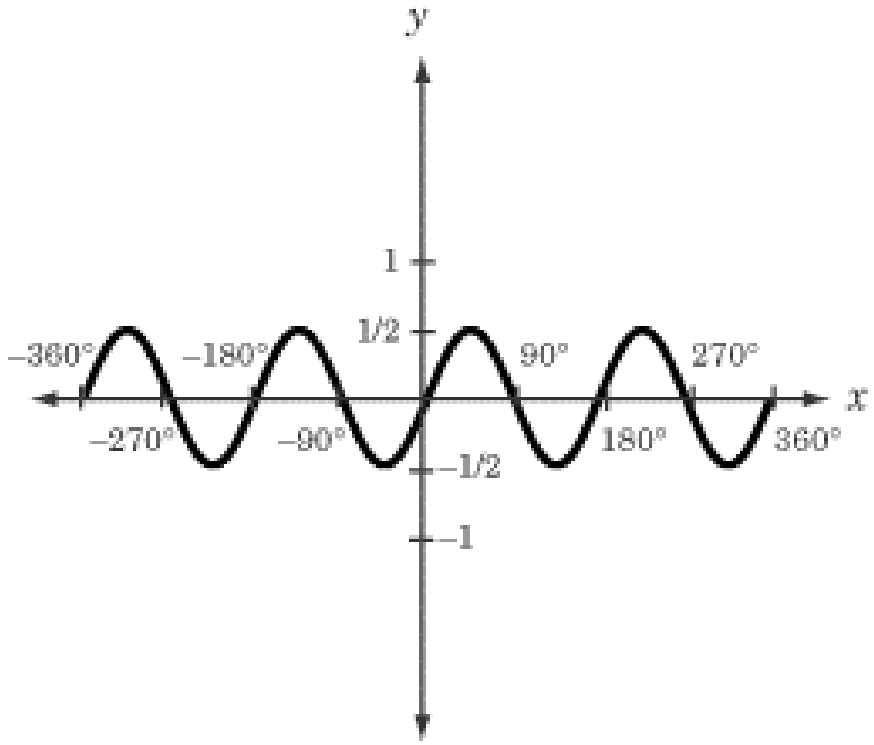
Example:

What is the period and amplitude of the function $y = 4\ sin3x$?

Period and Amplitude

Example:

What is the period and amplitude of the function graphed below?



Inverse Trigonometric Functions

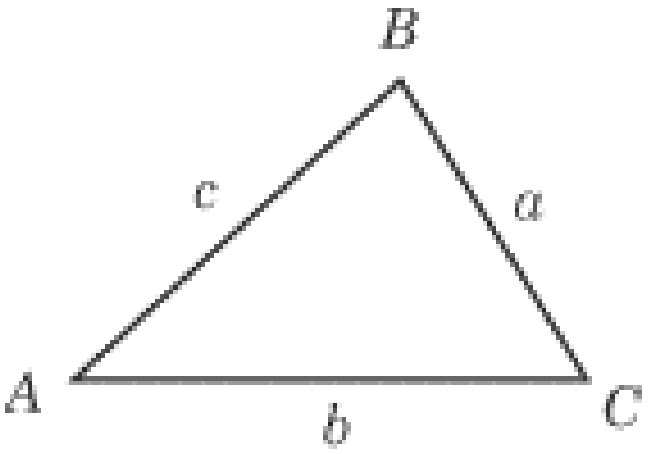
Example:

What angle between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ has a tangent of -1?

Solving Non-Right Triangles

Example:

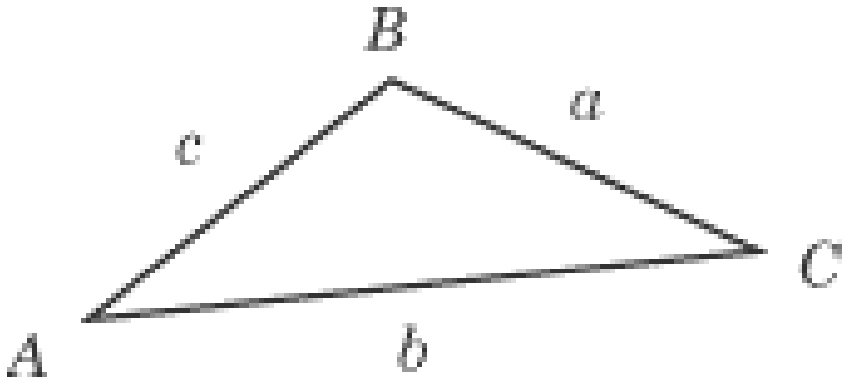
In triangle ABC, a = 5, b = 6, and $< B = 65°$. Solve the triangle.



Solving Non-Right Triangles

Example:

Solve triangle ABC if a = 4, b = 7, and c = 10.



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Miscellaneous Topics

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Statistical Analysis

Example:

If the average of four numbers is 22 and three of the numbers are 7, 11, and 18, then what is the fourth number?

Statistical Analysis

Example:

The mean age of the 14 numbers of a ballroom dance class is 34. When a new student enrolled, the mean age increased to 35. How old is the new student?

Probability

Example:

Joe has 3 green marbles, 2 red marbles, and 5 blue marbles. If all the marbles are dropped into a dark bag, what is the probability that Joe will pick out a green marble?

Probability

Example:

A teacher keeps a jar full of different flavored jelly beans on her desk and hands them out randomly to her class. But one particularly picky student likes only the licorice-flavored ones. If the jar has 50 beans in all—15 licorice, 10 cherry, 20 watermelon, and 5 blueberry—what is the probability that the first three jelly beans picked out are licorice flavored?

Permutations and Combinations

Example:

At a dog show, three awards are given: best in show, first runner-up, and second runner-up. A group of 10 dogs are competing in the competition. In how many ways can the awards be distributed?

Permutations and Combinations

Example:

Suppose that a committee of 10 people must elect three leaders, whose duties are all the same. In how many ways can this be done?

Group Questions

Example:

In a particular school, the school band has 42 members, and the school orchestra has 35 members. Seven students play in both the band and the orchestra, and 231 students play in neither the band nor the orchestra. How many students are in this particular school?

Group Questions

Example:

A room contains 80 people. Thirty have curly hair, 24 have blond hair, and 40 have hair that is neither curly nor blond. How many people in the room have curly, blond, hair?

Logic

Example:

The statement “If Jill misses the bus, she will be late” is true. Which other statement must be true?

(a) If Jill does not miss the bus, she will not be late

(b) If Jill is not late, she missed the bus

(c) If a student misses the buss, he or she will be late

(d) Jill is late because she missed the bus

(e) If Jill is not late, she did not miss the bus

Logic

The only must be true statement that can be derived from a premise.

All apples are fruit. $A \rightarrow F$

Can you conclude?

~~$A \rightarrow F$~~

~~$F \rightarrow A$~~

$F \rightarrow A$

Logic

Example:

What is the contrapositive of “Every book on the shelf is old”?

Arithmetic Sequences

- $a_n = a_1 + (n-1)d$
- n is the nth term in the sequence
- d is the difference between consecutive terms
- Sum of the first n terms

$$n\frac{a_1+a_n}{2}$$

Example:

If $a_4 = 4$ and $a_7 = 10$, find d.

Geometric Sequences

- $b_x = b_1, b_1 r, b_1 r^2, b_1 r^3 \ldots$
- Finding the nth term

$$b_n = b_1 r^{n-1}$$

- Sum of the first n terms

$$b_1 \frac{1 - r^n}{1 - r}$$

- Sum of an infinite geometric sequence

$$\frac{b_1}{1 - r}$$

Example:

What is the sum of the sequence 4, 2, 1, ½, ¼, …?

Limits

Example:

What is the limit of $f(x) = \frac{x+2}{x^2-2x-8}$ as x approaches -2?

Imaginary Numbers

- Represent the even roots of negative numbers
- $i = \sqrt{-1}$
- The powers of i work in cycles of four:
  - $i^1 = i$
  - $i^2 = \sqrt{-1} \times \sqrt{-1} = -1$
  - $i^3 = \sqrt{-1} \times \sqrt{-1} \times \sqrt{-1} = -i$
  - $i^4 = \sqrt{-1} \times \sqrt{-1} \times \sqrt{-1} \times \sqrt{-1} = 1$
- Evaluate $i^2 \times i^9$.

Complex Numbers

Example:

Simplify the expression $(3x + i)(x - 2i)$.

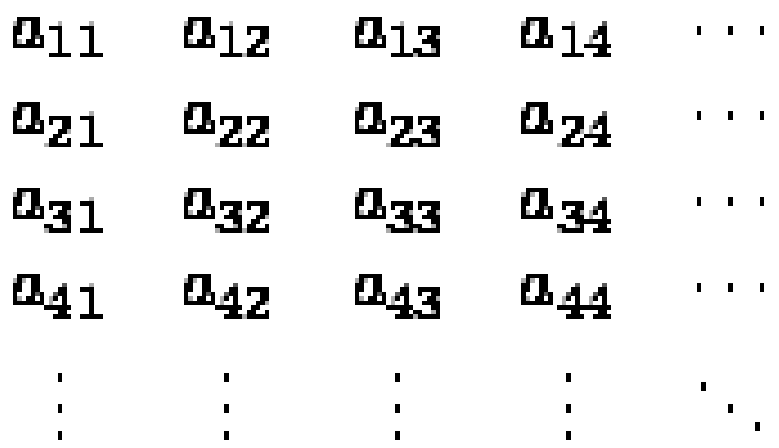
Complex Numbers

- The complex number a + bi is plotted in the complex plane exactly where the point (a, b) would be plotted on the coordinate plane
- The magnitude, or absolute value, of a complex number is the distance from the origin to that number in the complex plane
- Magnitude of a + bi (or |a + bi| = $\sqrt{a^2 + b^2}$

Example:

What is the magnitude of the complex number 4 – 9i?

Matrices



- Two matrices can be added if and only if they have the same number of rows and same number of columns → add corresponding entries
- Same for subtraction

Matrices

- To multiply a matrix by a scalar (a single constant, variable, or expression) multiply all the entries in the matrix by the scalar

$$5\begin{bmatrix} -4 & 3x \\ 12 & -\frac{1}{5} \\ 5x^2 & x^2-2x \\ \frac{3}{10} & \frac{1}{2}x \end{bmatrix} = \begin{bmatrix} -20 & 15x \\ 60 & -1 \\ 25x^2 & 5x^2-10x \\ \frac{3}{2} & \frac{5}{2}x \end{bmatrix}$$

Example:

What is 2A?

$$A = \begin{bmatrix} 4 & 3x & 5-x \\ -1 & y & -12 \end{bmatrix}$$

$$B = \begin{bmatrix} x & -16+x & \frac{x}{2} \\ 4 & x-y & -y \end{bmatrix}$$

Matrices

- To multiply two matrices, multiply each row in the first matrix by each column in the second matrix. Place the result of multiplying the 1st row by the 1st column in the 1st row and 1st column of the answer matrix. Place the result of multiplying the 1st row by the 2nd column in the 1st row and second column of the answer matrix. In general, place the result of multiplying row *i* by column *j* in row *i*and column *j* of the answer matrix.

$$\begin{bmatrix} 1 & 6 & -2 \\ 0 & -3 & 10 \end{bmatrix} \begin{bmatrix} 5 & 0 & -1 & 1 \\ 2 & 4 & 0 & 6 \\ -1 & -2 & 4 & \frac{1}{2} \end{bmatrix} = ?$$

$$\begin{bmatrix} 19 & 28 & -9 & 36 \\ -16 & -32 & 40 & -13 \end{bmatrix}.$$

Matrices

Example:

What is 3A + 2B?

$$A = \begin{bmatrix} 4 & 3x & 5-x \\ -1 & y & -12 \end{bmatrix}$$

$$B = \begin{bmatrix} x & -16+x & \frac{x}{2} \\ 4 & x-y & -y \end{bmatrix}$$